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# Homogeneous cosmological models in Yang's gravitation theory

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**Abstract.** We present a dynamic, spatially homogeneous solution of Yang's pure space gravitational field equations which is non-Einsteinian. The predictions of this cosmological model seem to be at variance with observations.

Yang (1974) presented a gauge theory of gravitation which is a rederivation, in vacuo, of the equations $M_{ijk} = R_{ij;k} - R_{ik;j} = 0$ first proposed by Kilmister and Newman (1961). This skew-symmetric combination of covariant derivatives of the Ricci tensor set to zero, called pure space equations by Yang, has received wide attention and has been considered recently by Barrett *et al* (1977). They discussed various aspects of the Kilmister–Yang (KY) equations with the apparent purpose of laying the foundations for new arguments in favour of the theory. Their analysis was not concerned with several unanswered criticisms (Pavelle 1975, 1976) which have been raised concerning unphysical characteristics of solutions of the equations. In view of this, it seems worthwhile to criticise further the KY equations by examining homogeneous dynamic cosmological models which predict further unphysical behaviour. Most of the earlier difficulties have been concerned with static models only (Pavelle 1976).

We choose an isotropic metric of the form

$$ds^2 = dt^2 - A(r, t)(dr^2 + r^2\, d\Omega^2) \tag{1}$$

and using MACSYMA† compute the very complicated nonlinear system of third-order equations. To simplify the resulting equations we assume the function $A(r, t)$ is separable as $A(r, t) = P(r) * Q(t)$. It is then possible to show that the component $M_{122}$ has a solution when $P = r^{-4}$. With this relation we find the KY system collapses to two equations for $Q(t)$ which are

$$QQ_{ttt} - Q_t Q_{tt} = 0 \tag{2a}$$

$$3Q_{tt} + CQ = 0 \tag{2b}$$

where $C$ is the constant scalar curvature. It is clear that (2*b*) satisfies (2*a*) and thus the following is our new solution of the KY equations:

$$ds^2 = dt^2 - Q_0 \exp[t * (-\tfrac{1}{3}C)^{1/2}] r^{-4} (dr^2 + r^2\, d\Omega^2). \tag{3}$$

† MACSYMA is the symbolic manipulation system of the Laboratory for Computer Science, Massachusetts Institute of Technology.

This may be transformed to the form

$$ds^2 = dt^2 - Q \exp[t * (-\tfrac{1}{3}C)^{1/2}](dr^2 + r^2\, d\Omega^2). \qquad (4)$$

A similar metric has been found as a cosmological solution in a modified version of Yilmaz theory (Tupper 1971). It is also like the metric in the steady-state theory (Adler *et al* 1975).

The value of the scalar curvature $C$ determines the large-scale behaviour of the solution. For $C > 0$ the universe is an oscillating one while for $C < 0$ it expands forever. The latter case does not correspond to an Einstein space. The observations in the Yang universe model for $C < 0$ would be very similar to those in the steady-state universe. For $C > 0$ we have a curious mixture of flat spatial sections with a closed universe. For real solutions we can arrange that $Q = Q_0 \cos(Kt)$ where $K = (-\tfrac{1}{3}C)^{1/2}$. Then the universe oscillates in volume with the evolutionary period $2\pi(-3/C)^{1/2}$. Present observational evidence seems to indicate an open universe (the $C < 0$ case) (Tinsley 1977). Nevertheless, observations would seem to be inconsistent with either choice of $C$ and of course for $C = 0$ we have only Minkowski space. It is interesting to note that there is an identical solution in the Einstein theory when the real cosmological constant $\lambda$ is present. The Einstein equations then read $G_{ij} = \lambda g_{ij}$ for the case $C < 0$ (McVittie 1965). However, when $C > 0$ there is no analogous Einstein space.

Barrett *et al* (1977) have presented a cosmological solution in the Yang theory, but it is not dynamic and bears no relation to the cosmological models usually considered in Einstein spaces. The model presented here is a dynamic expanding universe which can be submitted to the usual observational tests as made by McVittie (1965). Also, there is the peculiar $C > 0$, flat universe of oscillating space volume which we have exhibited.

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